(MAT4S) (2110-4)

B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2017.

Second Year — Fourth Semester

Part II — Mathematics

Paper IV — REAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following questions.

(Short answer questions).

1. Show that :

$$\lim\left[\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\ldots+\frac{1}{(n+n)^2}\right]=0.$$

2. Test for convergence of

(a) $\sum\frac{1}{n^3}\left(\frac{n+2}{n+3}\right)^n$

(b) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\ldots.$

3. Test for convergence $\sum\frac{2^n-2}{2^n+1}x^n (x>0)$.

4. Show that $f: R \to R$ defined by $f(x) = x$ if $x \in R - Q$ and $f(x) = -x$ if $x \in Q$ is continuous only at '0'.

5. Determine the constants a, b so that the function f defined by $f(x) = 2x + 1$ if $x \leq 1$; $f(x) = ax^2 + b$ if $1 < x < 3$; $f(x) = 5x + 2a$ if $x \geq 3$ is continuous every where.

6. If $f : [a,b] \to R$ and $g : [\alpha, \beta] \to R$, $I_m\ f \leq [\alpha, \beta]$, f is derivable at c and g is derivable at $f(c)$, then $g \circ f$ is derivable at c and $(g \circ f)'(c) = g'[f(c)]f'(c)$.

7. Find 'C' of cauchy's mean value theorem for $f(x) = \sqrt{x}$ and $g(x) = \dfrac{1}{\sqrt{x}}$ in $[a,b]$ where $0 < a < b$.

8. Prove that the function defined on $[0,1]$ defined below is not R- integrable,

$f(x) = 1$, when x is rational

$= -1$, when x is irrational

SECTION B — (5 × 10 = 50 marks)

Answer ALL of the following questions.

(Essay questions)

9. (a) (i) Show that the sequence $\{S_n\}$ defined by $S_1 = \sqrt{2}$, $S_{n+1} = \sqrt{2S_n}$ converges to 2.

(ii) Show that sequence $\{S_n\}$ defined by $S_1 = 1$ and $S_{n+1} = \sqrt{2 + S_n}$ for all $n \in N$ is convergent.

Or

(b) (i) Prove that every cauchy sequence is convergent.

(ii) If $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ then show that $\{S_n\}$ is not convergent by using Cauchy's general principle of convergence.

10. (a) Prove that $\sum u_n$ be a series of positive terms such that $\lim \frac{u_{n+1}}{u_n} = l$. Then (i) $\sum u_n$ converges if $l < 1$ (ii) $\sum u_n$ diverges if $l > 1$ and (iii) the test fails to decide the nature of the series of $l = 1$.

Or

(b) (i) Test the convergence and absolutely convergence of the series $\sum \frac{(-1)^{n+1} n}{n^2 + 1}$.

(ii) Show that the series $\sum (-1)^n \left[\sqrt{n^2 + 1} - n\right]$ is conditionally convergent.

11. (a) Prove let $f : [a : b] \to R$. If f is continuous on $[a, b]$ then f attains its exact bounds atleast once in $[a, b]$.

Or

(b) (i) Let $f: R \to R$ be defined by $f(x) = \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ if $x \neq 0$ and $f(0) = 1$. Show that f has a discontinuity of the first kind at $x = 0$.

(ii) Examine for continuity of the function f defined by $f(x) = |x| + |x-1|$ at $x = 0$ and 1.

12. (a) State and prove Cauchy's mean value theorem.

Or

(b) Show that $x - \dfrac{x^2}{x} < \log(1+x) < x - \dfrac{x^2}{2(1+x)}$ for $x > 0$.

13. (a) Show that $f(x) = 3x + 1$ is integrable on $[1,2]$ and $\int_1^2 f(x)\,dx = \dfrac{11}{2}$.

Or

(b) (i) Show that $\dfrac{1}{\pi} \leq \int_0^1 \dfrac{\sin \pi x}{1+x^2}\,dx \leq \dfrac{2}{\pi}$.

(ii) Prove that $\dfrac{\pi^3}{24} \leq \int_0^{\pi} \dfrac{x^2}{5 + 3\cos x}\,dx \leq \dfrac{\pi^3}{6}$.